Chapter 2

An Introduction to Radio Astronomy Observables

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An introduction to some basic observables in radio astronomy is presented. These include flux density, intensity/brightness, brightness temperature, and some antenna fundamentals (power pattern, gain, antenna temperature, etc.). The concept of the radiative transfer and its application in radio astronomy is also briefly explained.

I have restricted the text to presenting the fundamentals. For those who are interested, please refer to the textbooks listed in the end for a more in-depth reading.

2.1 Introduction

In general, radiation from a source can be described by four Stokes parameters (I, Q, U, and V) as a function of frequency (ν), time (t), and its position (α and δ). The parameters for linear polarization (Q and U) and circular polarization (V) carry important information on the magnetic field structure in and around the source. But mainly because of the intrinsic weakness of detectable polarization from most of the sources, the measurements of these three Stokes parameters are relatively limited compared to the observations of the total intensity (I). Therefore, the lecture will focus on the randomly polarized (or, unpolarized) radiation. Radio waves from most of the astronomical sources are known to be practically unpolarized.

2.2 Flux Density (Flux) versus Intensity (Brightness)

Two most widely used quantities in radio astronomy are *spectral flux density* and *specific intensity*.

For brevity, the spectral flux density is often referred to as *flux density* or even *flux*, S_ν. It is defined

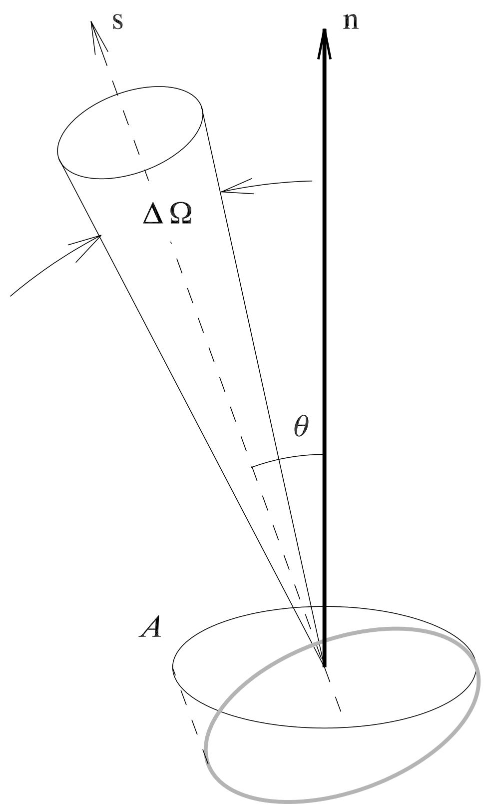

Figure 2.1: A schematic geometry used in the definition of intensity (brightness)

as the power (ΔW) received (or transmitted) within a certain frequency interval ($\Delta\nu$) and through a certain area (A), i.e.,

$$S_\nu = \frac{\Delta W}{A\Delta\nu} \; . \tag{2.1}$$

And thus it has SI units of W m^{-2} Hz^{-1}. Because radio source is usually very weak, a more frequently used unit in radio (and infrared) astronomy to measure flux density is *jansky* (abbreviated Jy) and

$$1 \text{ Jy} = 10^{-26} \text{ W m}^{-2} \text{ Hz}^{-1} = 10^{-23} \text{ ergs cm}^{-2} \text{ Hz}^{-1}. \tag{2.2}$$

The specific intensity usually appears as *intensity*, I_ν, and is interchangeable to the so-called *(surface) brightness*, B_ν, in optical astronomy. It can be understood as the flux density per unit solid angle ($\Delta\Omega$). A more general definition of I_ν or B_ν is

$$I_\nu = B_\nu = \frac{\Delta W}{\cos\theta A\Delta\nu\Delta\Omega} \; , \tag{2.3}$$

here θ, as shown in Figure 2.1, is the angle between the normal to the area (A) and the direction to the solid angle ($\Delta\Omega$). Obviously, it has SI units of W m^{-2} Hz^{-1} sr^{-1}.

It should be noted that flux density S_ν is usually applied to point sources while (surface) brightness B_ν or intensity I_ν applied to extended sources. Assuming that the surface of any extended source is composed of many infinitesimal areas, and each area is so small that it has a constant surface brightness at that particular position in the sky. Thus, the overall flux density of the extended source is

$$S_\nu = \int_{\mathrm{src}} I_\nu \cos\theta \mathrm{d}\Omega \ . \tag{2.4}$$

In spherical polar coordinates, we have the differential element of solid angle $\mathrm{d}\Omega = \sin\theta \mathrm{d}\theta \mathrm{d}\phi$. Here, ϕ is the azimuthal angle and θ the polar angle.

It can be proved that the intensity I_ν (or brightness B_ν) is independent of the distance (r) of the emitting object. However, the flux density S_ν is proportional to $1/r^2$.

2.3 Black Body Radiation and Brightness Temperature

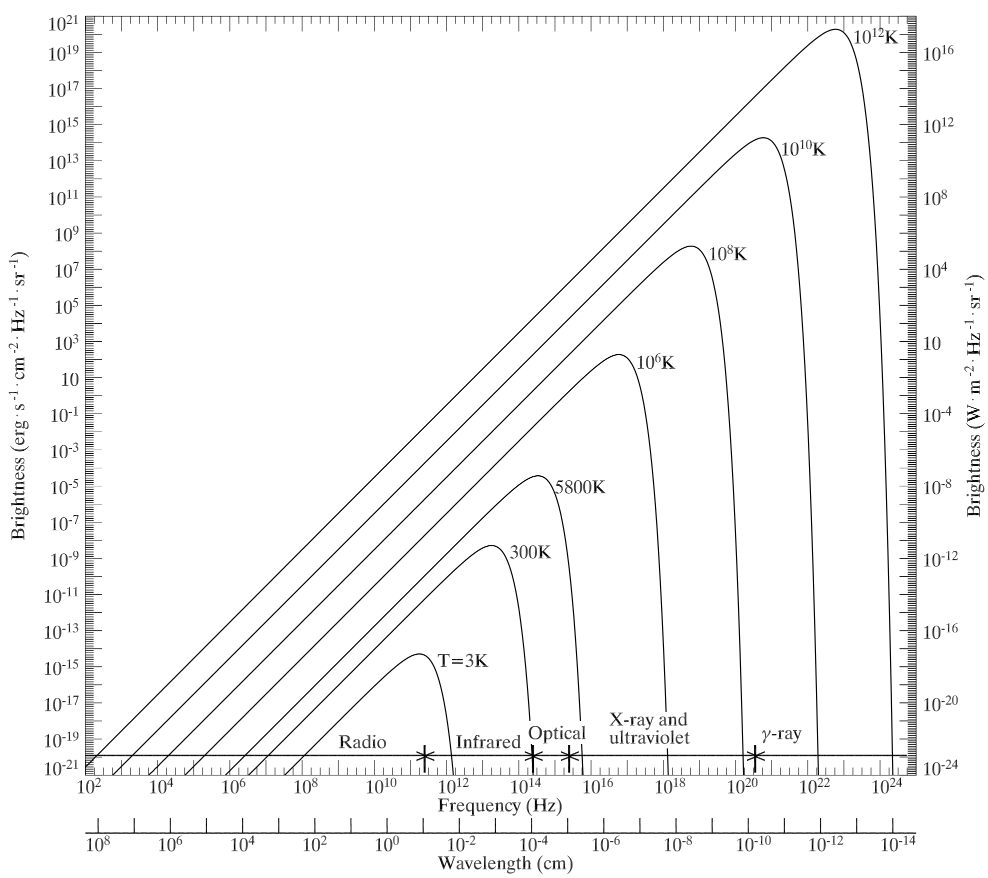

Figure 2.2: Planck function for a wide range of temperatures. Each curve is determined by a particular (labelled) temperature for black body radiation.

The spectral distribution of a black body radiator in thermodynamic equilibrium is given by the well-known Planck function:

$$B_\nu = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1} \ , \tag{2.5}$$

where $h = 6.63 \times 10^{-34}$ J s is Planck's constant, $k = 1.38 \times 10^{-23}$ J K^{-1} is Boltzmann's constant, and $c = 3 \times 10^8$ m s^{-1} is the speed of light. As shown in Figure 2.2, temperature T is the only parameter needed to describe such a Planck spectrum of black body radiation. At a given temperature, the black body spectrum has a maxima brightness with the corresponding frequency in GHz (10^9 Hz)

$$\frac{\nu_{\rm max}}{\rm GHz} = 58.789[\frac{T}{\rm K}] \ , \tag{2.6}$$

or the corresponding wavelength

$$\frac{\lambda_{\rm max}}{\rm cm} = 0.28978[\frac{T}{\rm K}]^{-1} \ . \tag{2.7}$$

This is known as *Wien's displacement law*, implying that a black body appears bluer as T increases.

Integrating the Planck function over the entire frequency range, we can obtain the *Stefan-Boltzmann law* as follows,

$$I(T) = B(T) = \frac{1}{\pi}\sigma T^4 \ , \tag{2.8}$$

where σ is the Stefan-Boltzmann constant:

$$\sigma = \frac{2\pi^5 k^4}{15c^2h^3} = 5.6697 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4} \ . \tag{2.9}$$

We introduce two extreme cases of Planck function. In the case where the exponential becomes much greater than unity, i.e.,

$$h\nu/kT >> 1 \ , \qquad \text{or} \qquad e^{h\nu/kT} >> 1 \ , \tag{2.10}$$

we can obtain the *Wien distribution* as

$$B_\nu = \frac{2h\nu^3}{c^2} e^{-h\nu/kT} \tag{2.11}$$

which is an appropriate approximation when the temperature is low and the frequencies are high. In the opposite case where the value of the exponential is much less than unity, i.e.,

$$h\nu/kT << 1 \ , \qquad \text{or} \qquad e^{h\nu/kT} - 1 << 1 \ , \tag{2.12}$$

the Planck function reduces to the *Rayleigh-Jeans distribution*

$$B_\nu = \frac{2kT\nu^2}{c^2} = \frac{2kT}{\lambda^2} \ . \tag{2.13}$$

This works at high temperature and low frequencies. Plugging in constants h and k, we can show that the Rayleigh-Jeans limit holds for frequencies

$$\frac{\nu}{\rm GHz} << 20.84[\frac{T}{\rm K}] \ , \qquad \text{or} \qquad \nu << 0.3545 \ \nu_{\rm max} \ , \tag{2.14}$$

where $\nu_{\rm max}$ is estimated from the Wien's displacement law at a given temperature T. This is a very good approximation in the radio frequency range. In particular, this actually defines a very useful quantity,

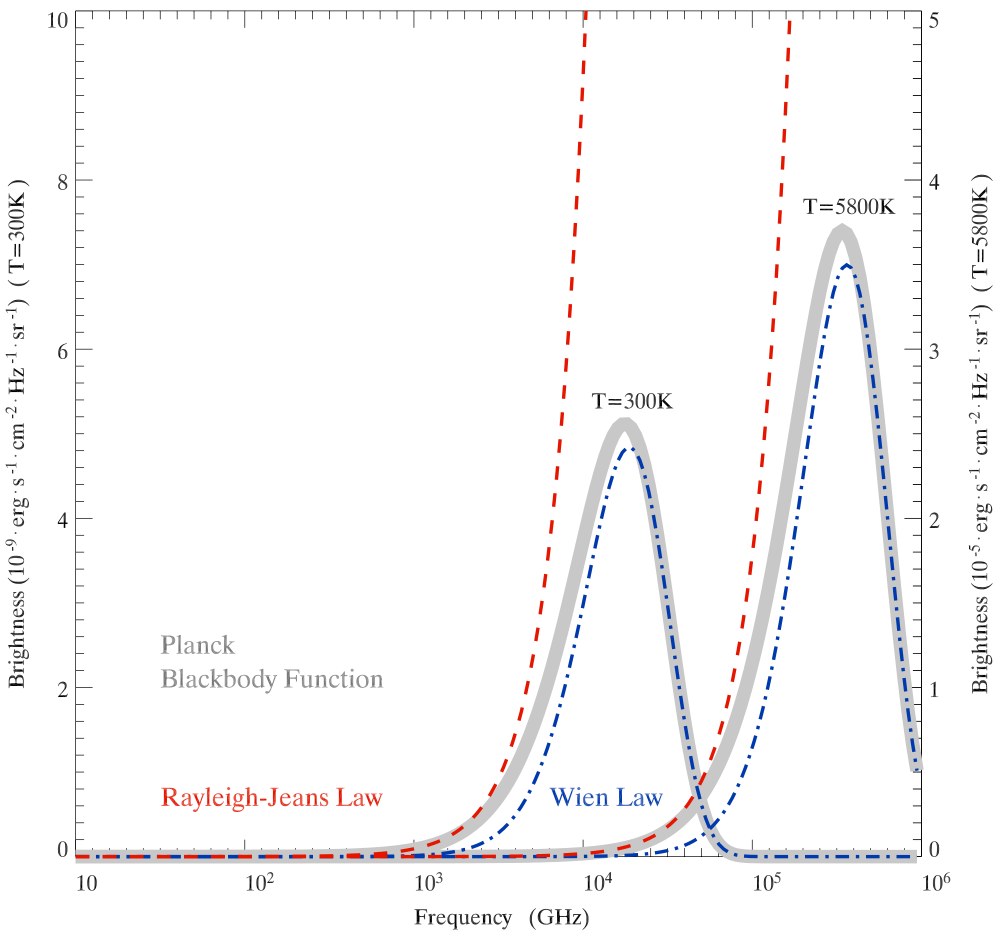

Figure 2.3: Planck function versus two extreme cases: Rayleigh-Jeans approximation (red dashed lines) and Wien approximation (blue dot-dashed lines). Note the difference in the vertical axis (left for brightness at T=300 K and right at T= 5800 K).

the *brightness temperature* $T_{\rm b}$, in radio astronomy to measure the distribution of the surface brightness from any radiation mechanism,

$$T_{\rm b} = \frac{c^2}{2k\nu^2} B_\nu \ . \tag{2.15}$$

Obviously, if the source is a black body with a temperature T and the Rayleigh-Jeans law applies, the brightness temperature $T_{\rm b}$ must correspond to the thermodynamic temperature, i.e., $T_{\rm b} = T$. Otherwise, $T_{\rm b}$ has little to do with the physical temperature. Unlike the thermodynamic temperature which is always independent of frequency, the brightness temperature of the radiation other than the black body radiation is a function of frequency. Therefore, in most cases of radio astronomy, $T_{\rm b}$ is an equivalent temperature, not a physical temperature! It tells what the temperature of the source would have to be if it were radiating like a black body in order to display the observed surface brightness at a given frequency. Note that in some cases such as at millimeter and sub-millimeter wavelengths, the Rayleigh-Jeans law is no longer applicable. Then, the defined brightness temperature should be calculated from

$$T_{\rm b} = \frac{h\nu}{k}\frac{1}{e^{h\nu/kT_{\rm b}}-1} \ . \tag{2.16}$$

2.4 Calculations of Brightness Temperature

We can write down a general relation between the flux density of a source and the corresponding brightness temperature distribution $T_{\rm b}(\theta,\phi)$ across the sky:

$$S_\nu = \frac{2k\nu^2}{c^2}\int T_{\rm b}(\theta,\phi){\rm d}\Omega \ . \tag{2.17}$$

In the following, I will show how to use this expression to calculate the brightness temperature for different source geometries.

For the uniform (constant) brightness temperature distribution, we have $T_{\rm b}(\theta,\phi)= T_{\rm b}$=constant and $\int_{\rm src}{\rm d}\Omega = \frac{\pi}{4}\theta^2$, where θ is the source size in diameter. Thus, we obtain

$$\frac{T_{\rm b}}{10^{12}{\rm K}} = 1.76[\frac{S_\nu}{1{\rm Jy}}][\frac{\nu}{1{\rm GHz}}]^{-2}[\frac{\theta}{1{\rm mas}}]^{-2} \ . \tag{2.18}$$

This corresponds to an optically-thick sphere distribution. More realistically, some distributions of brightness temperature over the extended source should be considered. Assuming a Gaussian brightness temperature distribution of

$$T_{\rm b}(\theta) = T_0{\rm e}^{-\frac{(\theta/2)^2}{2\sigma^2}} \ , \tag{2.19}$$

here T_0 is the peak brightness temperature and $\sigma = \frac{\theta^2_{\rm FWHM}}{8{\rm ln}2}$ ($\theta_{\rm FWHM}$ is the Full Width at Half Maximum (FWHM) of the radio source measured from high-resolution radio observations), we can derive

$$\frac{T_0}{10^{12}{\rm K}} = 1.22[\frac{S_\nu}{1{\rm Jy}}][\frac{\nu}{1{\rm GHz}}]^{-2}[\frac{\theta_{\rm FWHM}}{1{\rm mas}}]^{-2} \ . \tag{2.20}$$

Similarly, for the optically-thin sphere distribution (the brightness temperature $T_{\rm b}(\theta)$ at each point is proportional to the path length through the sphere of $\theta_{\rm F}$ in its angular diameter),

$$T_{\rm b}(\theta) = T_0\sqrt{1-(\frac{\theta}{\theta_{\rm F}})^2} \qquad (0 \leq \theta \leq \theta_{\rm F}) \ , \tag{2.21}$$

here T_0 is the brightness temperature at the center of the sphere, we then obtain

$$\frac{T_0}{10^{12}{\rm K}} = 2.64[\frac{S_\nu}{1{\rm Jy}}][\frac{\nu}{1{\rm GHz}}]^{-2}[\frac{\theta_{\rm F}}{1{\rm mas}}]^{-2} \ . \tag{2.22}$$

It is interesting to compare the brightness temperatures of the above-mentioned three distributions

$$(T_0)_{\rm sphere} : (T_{\rm b})_{\rm constant} : (T_0)_{\rm Gaussian} = \frac{3}{\theta^2_{\rm F}} : \frac{2}{\theta^2} : \frac{2{\rm ln}2}{\theta^2_{\rm FWHM}} \ , \tag{2.23}$$

here the same flux density at the same frequency is assumed.

2.5 Antenna Fundamentals

The first radio telescope (antenna) was built in 1937 by Grote Reber. Basically, antenna is such a device that can receive the electromagnetic wave at radio frequency. Following the previous discussion, we can think of an antenna as a scanner of the brightness or the brightness temperature of the sky. One of the most important parameters of an antenna is its power of resolving the celestial object, the angular resolution, which is of the order of $\sim \lambda/\mathrm{D}$, here D is the diameter of the antenna. This means that antenna is directive; every time it is only sensitive to the signals within a certain angular dimension. More generally, this can be characterized by the *normalized power pattern* as a function of the direction θ and ϕ, $P(\theta,\phi)$, normalized so that $P(0,0)=1$ in the direction of the symmetry axis and otherwise, $P(\theta,\phi) \leq 1$. Therefore, let a point-like source be in a direction $(\theta,\phi) \neq (0,0)$, the received power would be

$$\Delta W = \frac{1}{2}\mathrm{A}_e S_\nu P(\theta,\phi)\Delta\nu \ , \tag{2.24}$$

here the factor 1/2 is introduced to account for the fact that only one polarization can be detected, and A_e is an effective aperture of the antenna. The effective collecting area is smaller than the geometrical area A_g, and the ratio between the two gives the definition of the *aperture efficiency* as

$$\eta_A = \frac{A_e}{A_g} < 1 \ . \tag{2.25}$$

An extended radio source with a brightness distribution $B(\theta,\phi)$ or $T_\mathrm{b}(\theta,\phi)$, can be treated as the collection of many point sources. Thus, the total power received within a certain solid angle is

$$W = \frac{1}{2}\mathrm{A}_e \int_{\mathrm{sky}} B(\theta,\phi)P(\theta,\phi)\mathrm{d}\Omega\Delta\nu \ , \tag{2.26}$$

or,

$$W = \frac{k}{\lambda^2}\mathrm{A}_e \int_{\mathrm{sky}} T_\mathrm{b}(\theta,\phi)P(\theta,\phi)\mathrm{d}\Omega\Delta\nu \ . \tag{2.27}$$

Before proceeding, we introduce two solid angles. The *beam solid angle* Ω_A of an antenna is a measure of the angular extent of the antenna beam:

$$\Omega_\mathrm{A} = \int_{\mathrm{sky}} P(\theta,\phi)\mathrm{d}\Omega \ . \tag{2.28}$$

The *main beam solid angle* Ω_M is similar to Ω_A but has the integration over the main lobe only:

$$\Omega_\mathrm{M} = \int_{\mathrm{main\ lobe}} P(\theta,\phi)\mathrm{d}\Omega \ . \tag{2.29}$$

And the ratio of the power in main lobe to the total power defines the *(main) beam efficiency*

$$\eta_B = \frac{\Omega_\mathrm{M}}{\Omega_\mathrm{A}} < 1 \ . \tag{2.30}$$

Furthermore, we define another term called the *maximum directive gain* G_max or *directivity* D as

$$G_\mathrm{max} = D = \frac{4\pi}{\Omega_\mathrm{A}} \ . \tag{2.31}$$

This is to measure the gain of a real directional antenna relative to an idealized isotopic antenna (which has a beam solid angle of 4π), and is usually calculated in decibels (dB) as $D(\mathrm{dB}) = 10\log_{10}(4\pi/\Omega_\mathrm{A})$. In practice, the antenna effective area A_e and thus the beam solid angle Ω_A and gain G_max are a function of elevation angle. Most antennas are set to have maximum efficiency at an elevation angle of $45°$. Therefore *gain curve*, as a plot of gain versus elevation angle, is necessary to show the performance of an antenna.

Another important term in radio astronomy is called *antenna temperature* T_A. It is equivalent to the power received within a certain frequency interval, but in the units of temperature (K). According to the *Nyquist theorem* which identifies the power stimulated by the presence of a radio source with that produced by heating the characteristic antenna resistance to a temperature T_A, we have

$$T_A = \frac{W}{k\Delta\nu} \ . \tag{2.32}$$

It should be mentioned here that in reality, the stimulated power includes everything (radio source, sky, ground spill-over, etc.) received within the antenna beam, and thus T_A is not solely related to the source. Obviously, T_A has nothing to do with the physical temperature of the antenna itself. We can derive T_A in terms of the sky brightness temperature $T_\mathrm{b}(\theta, \phi)$

$$T_A = \frac{A_e}{\lambda^2} \int_\mathrm{sky} T_\mathrm{b}(\theta, \phi) P(\theta, \phi) \mathrm{d}\Omega \ . \tag{2.33}$$

Considering a simple case which T_b is constant and thus $T_A = T_\mathrm{b}$, we can get a very powerful relation between the aperture A_e and the beam solid angle Ω_A as follows:

$$A_e \Omega_\mathrm{A} = \lambda^2 \ . \tag{2.34}$$

This holds for more general case and any antenna. This can be understood as an antenna equivalent of the angular resolution $\Theta \propto \lambda/\mathrm{D}$ (where, $\Omega_\mathrm{A} \propto \Theta^2$ and $A_e \propto \mathrm{D}^2$). This important equation closely relates two different concepts: the concept of the power *gain* (G_max or D) of a transmitting antenna through its beam solid angle (Ω_A) and that of *effective area* (A_e) of a receiving antenna. Now we can interpret the antenna temperature as the weighted mean brightness temperature at a certain direction (θ, ϕ) with the antenna (normalized) power pattern as the weight:

$$T_A = \frac{\int T_\mathrm{b}(\theta, \phi) P(\theta, \phi) \mathrm{d}\Omega}{\int P(\theta, \phi) \mathrm{d}\Omega} \ . \tag{2.35}$$

There is a *reciprocity theorem*, stating that parameters for a receiving antenna are the same as ones for a transmitting antenna. This can be proved based on the fact that the solutions of Maxwell's equations are valid when time is reversed.

2.6 Radiative Transfer

So far we have discussed about the radiation in free space, and viewed antenna as a region between a guided wave (electrical signal) and an electromagnetic wave from the radio emitter. However, when there is medium along the ray path, such as the neutral and ionized media lying between a radio source and the surface of the Earth, the energy received by an antenna will no longer be identical to the energy originally emitted by a radio source (as indicated in Figure 2.4). This can be understood in terms of the *radiative transfer* equation

$$\frac{\mathrm{d}I_\nu}{\mathrm{d}s} = \frac{\mathrm{d}I_\nu^+}{\mathrm{d}s} + \frac{\mathrm{d}I_\nu^-}{\mathrm{d}s} = j_\nu - \kappa_\nu I_\nu \ , \tag{2.36}$$

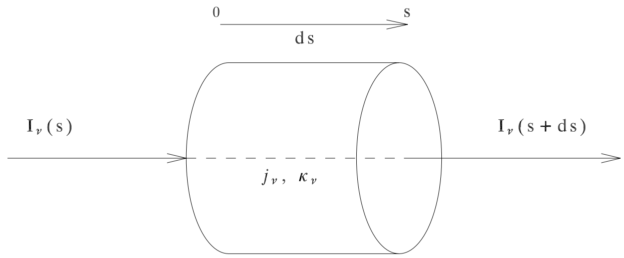

Figure 2.4: A schematic diagram used to illustrate the concept of radiative transfer

here the gain term,

$$\mathrm{d}I_\nu^+ = j_\nu \mathrm{d}s \ , \tag{2.37}$$

is described by the *(volume) emissivity* j_ν (radiation produced per unit volume and per unit solid angle within a certain frequency band) and the loss term here,

$$\mathrm{d}I_\nu^- = -\kappa_\nu I_\nu \mathrm{d}s \ , \tag{2.38}$$

is assumed to be caused by the absorption only (no scattering) with the *absorption coefficient* κ_ν. We can further define an important term, the *optical depth* τ_ν, by integrating $\mathrm{d}\tau_\nu = \kappa_\nu \mathrm{d}s$ over the ray path of radiation transfer

$$\tau_\nu(s) = \int_0^s \kappa_\nu(s')\mathrm{d}s' \ . \tag{2.39}$$

Immediately, there are two simple solutions to the radiative transfer equation. One is for the emission only (i.e. $\kappa_\nu = 0$). The equation of radiative transfer will be $\frac{\mathrm{d}I_\nu}{\mathrm{d}s} = j_\nu$ and, thus the solution is

$$I_\nu(s) = I_\nu(0) + \int_0^s j_\nu(s')\mathrm{d}s' \ , \tag{2.40}$$

here $I_\nu(0)$ is the intensity at the origin, corresponding to $s = 0$ along the ray path. The other is for the absorption only (i.e. $j_\nu = 0$). The radiative transfer equation will become $\frac{\mathrm{d}I_\nu}{\mathrm{d}s} = -\kappa_\nu I_\nu$, and we get

$$I_\nu(s) = I_\nu(0)\mathrm{e}^{-\tau_\nu(s)} \ . \tag{2.41}$$

In the case of thermodynamic equilibrium, i.e., I_ν=constant along any ray path or mathematically $\frac{\mathrm{d}I_\nu}{\mathrm{d}s} = 0$, both the emissivity j_ν and the absorption coefficient κ_ν are related through the intensity I_ν which is the Planck function $B_\nu(T)$,

$$\frac{j_\nu}{\kappa_\nu} = I_\nu = B_\nu(T) \ . \tag{2.42}$$

This is also know as *Kirchoff's Law*, and the ratio $(\frac{j_\nu}{\kappa_\nu})$ is also known as the *source function*. Beware that in general case, I_ν is different from $B_\nu(T)$. However, we can use the so-called Local Thermodynamic Equilibrium (LTE) approximation which satisfies the Kirchoff's Law, and the radiative transfer equation can be re-written as

$$-\frac{1}{\kappa_\nu}\frac{\mathrm{d}I_\nu}{\mathrm{d}s} = -\frac{\mathrm{d}I_\nu}{\mathrm{d}\tau_\nu} = I_\nu - B_\nu(T) \ . \tag{2.43}$$

Finally, with the assumption that temperature T is the constant along the ray path (independent of the optical depth τ_ν), we can derive a more general and very insightful solution as

$$I_\nu(s) = I_\nu(0)\mathrm{e}^{-\tau_\nu(s)} + B_\nu(T)(1 - \mathrm{e}^{-\tau_\nu(s)}) \ . \tag{2.44}$$

In radio astronomy, this is more often expressed in terms of the various temperatures,

$$T_\mathrm{b}(s) = T_\mathrm{b}(0)\mathrm{e}^{-\tau_\nu(s)} + T(1 - \mathrm{e}^{-\tau_\nu(s)}) \ , \tag{2.45}$$

where T_b and T correspond to the brightness temperature of a radio source and the thermodynamic temperature of the medium, respectively. The first term in the right hand side of above two equations represents the power (brightness temperature) of incoming radiation (from a radio source) attenuated by a factor $\mathrm{e}^{-\tau_\nu(s)}$, and the second term is the net contribution to the received power (temperature) from the intervening medium itself as a black body. Two interesting cases are: for $\tau_\nu(s) >> 1$ (optically thick), then $T_\mathrm{b}(s) = T$; for $\tau_\nu(s) << 1$ (optically thin), then $T_\mathrm{b}(s) = T_\mathrm{b}(0) + T\tau_\nu(s)$.

2.7 An Example - Opacity Correction

High frequency radio emission from a distant radio source ofter suffers the atmospheric absorption on its way to the ground antenna. As a result, any signal above the atmosphere could be heavily attenuated by the atmosphere that is characterized by temperature (T_atm) and optical depth (τ_atm). In terms of the *system temperature* T_sys (which is an input-equivalent temperature of the noise added into the receiving system), we have

$$T_\mathrm{sys} = T_\mathrm{rx} + T_\mathrm{atm}(1 - \mathrm{e}^{-\tau_\mathrm{atm}}) \ . \tag{2.46}$$

This means that the measured system temperature consists of the temperature of receiver T_rx and a contribution from the atmosphere. Note that in many case, T_sys is much larger than the previously mentioned antenna temperature T_A. With the assumption that the atmosphere is composed of a set of parallel planes, we can approximate the optical depth (which is proportional to the path length in the atmosphere) as

$$\tau_\mathrm{atm} = \tau_0 \mathrm{secZ} \ , \tag{2.47}$$

here τ_0 is the zenith opacity, i.e., the atmospheric optical depth at the zenith (Z=0, Z is the local zenith angle). And if the exponential term ($\tau_\mathrm{atm} = \tau_0\mathrm{secZ}$) is small enough (which is roughly satisfied in most radio observations), we can have the following linear expression of T_sys as a function of secZ,

$$T_\mathrm{sys} = T_\mathrm{rx} + T_\mathrm{atm}\tau_0 \mathrm{secZ} \ . \tag{2.48}$$

Therefore, we can obtain the product of τ_0 and T_atm and the receiver temperature (T_rx) from a straight line fit to the real data. Then, if we know the atmosphere temperature T_atm, we will be able to estimate the zenith opacity τ_0.

Figure 2.5 shows plots of the system temperature (T_sys) versus secZ at four antenna sites (LA, MK, OV, and PT) during an 86 GHz VLBA experiment on November 3, 2002. By performing a linear fit to these plots, we can obtain the receiver temperature and atmospheric opacity as follows:

$$T_\mathrm{rx} = 116\mathrm{K}, \quad \tau_0 = 0.097 \qquad (\mathrm{LA}) \ ,$$

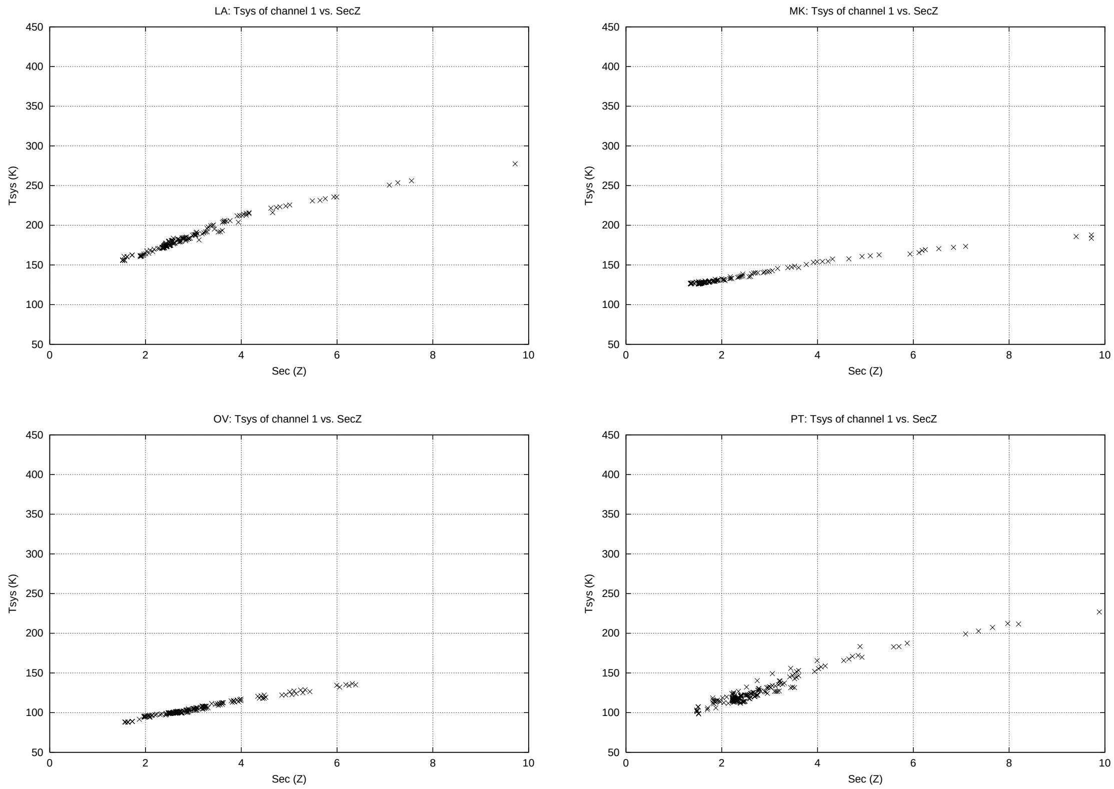

Figure 2.5: Plots of the system temperature ($T_{\rm sys}$) versus secZ at four antenna sites (LA, MK, OV, and PT) during an 86 GHz VLBA experiment on November 3, 2002 (Shen et al. 2005). These are used to determine the receiver temperature of each antenna and the optical depth of the atmosphere at each site.

$$T_{\rm rx} = 112{\rm K}, \quad \tau_0 = 0.041 \qquad ({\rm MK})\ ,$$
$$T_{\rm rx} = 73{\rm K}, \quad \tau_0 = 0.041 \qquad ({\rm OV})\ ,$$
$$T_{\rm rx} = 69{\rm K}, \quad \tau_0 = 0.081 \qquad ({\rm PT})\ ,$$

here an atmospheric temperature $T_{\rm atm} = 250{\rm K}$ is assumed.

For a better amplitude calibration, we usually need to refer the system temperature measured on the ground to a point above the atmosphere by multiplying $T_{\rm sys}$ by $e^{\tau_0 {\rm secZ}}$. This is referred to as the *opacity correction*, and the resultant modified system temperature $T^*_{\rm sys}$ is

$$T^*_{\rm sys} = T_{\rm rx} e^{\tau_0 {\rm secZ}} + T_{\rm atm}(e^{\tau_0 {\rm secZ}} - 1)\ . \tag{2.49}$$

L. Huang is thanked for preparing Figures 2.1, 2.2, 2.3, and 2.4.

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